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AMSAT-OSCAR-7: 50 Years in a Sun-Synchronous Orbit - And Still Counting -

Confirmation of the Long-Term Solar Gravity-Gradient Perturbation

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ABSTRACT

The first Delta launch vehicle to launch multiple secondary payloads was Δ-104. Carried, as the 2nd piggyback payload on that mission was an amateur radio Smallsat, AMSAT-OSCAR-7 (AO-7). The launch, on 15 November 1974, was to a 1460 km SSO. This spacecraft, as its name suggests, was the 7th in an on-going series of spacecraft operating in the Amateur Satellite Service. The amazing story associated with the mission of this small satellite is being reported elsewhere in these proceedings. While the particulars of this satellite's lifetime are exciting, we didn't particularly expect this "old bird's set of tricks to extend into our investigation of its orbit. We were particularly interested in determining when the spacecraft would be in 100% sun moving forward in time. To do this we investigated the orbit's history. We were surprised by the outcome. This paper describes our observations.

We utilized the information now available on Space-Track.com to look into this spacecraft's first 50 years of space flight history. This website "publishes" the, now famous, TLEs (two-line elements) dating back to 1958 when NORAD (now CSpOC) began its role of maintaining the catalog of all space objects. In order to discover if the satellite had progressed slowly forward in its sun-synchronous orbit as we had expected, we needed to track the sun angle (Φ) over time. To simplify the procedure of following the mean sun time of the orbit, we describe a method for sampling the AO-7 orbit RAAN value at Vernal Equinox of each year.

*Our search for the drift in our SSO led to the unexpected finding that both the elements RAAN (**Ω**) and inclination (**i**) exhibit a sinusoidal variation with a period of approximately 29 years. We further identified this to be caused by a solar gravity gradient torque, which perturbs LEO sun-synchronous orbits. We, here, provide in Appendix 2 the full analysis confirming that the solar gravity-gradient torque acting on the plane of the orbit is the cause of the observed oscillation. The differential equation we derived is essentially the same as that of the classical pendulum equation, which can be linearized to become a simple harmonic oscillator. While we initially believed we had discovered a new form of orbit perturbation, we've now uncovered an earlier finding of this phenomenon described in a NASA X-document by Ken Duck. Quite ironically, this discovery by K. Duck, a NASA/GSFC employee, used the ITOS series orbit to demonstrate his version of the solution to this differential equation. This is the same orbit into which AO-7 was injected. While we cannot, therefore, claim the discovery of this solar gravity-gradient perturbation along with the coupling of this effect with the Earth's J_2 perturbation, we are able to use 50 years of NORAD data to demonstrate that K. Duck's discovery is valid over nearly two cycles of the perturbation. And, we've verified this not only using the data for the still functional AO-7 spacecraft, but we've also verified this using the TLEs for the NOAA-4 spacecraft, the primary Δ-104 passenger (now non-functional); this spacecraft also experienced the same solar gravity-gradient perturbation.*

1.0 Introduction:

We've reported elsewhere in this Proceeding[1] regarding the unusual story of the small satellite, known as AMSAT-OSCAR-7 (AO-7). It was designed, fabricated, tested, and even launched by radio amateurs. And, radio amateur command and control stations also operate the satellite. And, its communication services were and still ARE – consumed by radio amateurs.

In November of 2024 AO-7 will celebrate its 50th anniversary in space. And, because this old satellite still lives and only because of that, this little spacecraft has saved it's best secret until now. This secret was in clear sight, embedded in NORAD/JSpOC/CSpOC published Two Line Element (TLE) data for all who might be interested to see. In November of 2023, it dawned on us to have a look at the precession of the orbit in more detail. The spacecraft operates only in sunlight due to a double failure of its NiCd battery[2] and we were motivated to determine the "drift" in the spacecraft's sun-synchronous orbit (SSO) in order to determine the percentage of in-sun vs. eclipse time for each orbit into the future. Had this satellite died when the NiCd batteries failed the first time, what we're about to report would never have been noticed. We believe it is fair to observe that there is little interest taken in dead satellite TLEs. Perhaps this paper could change this situation.

Let us first summarize the mission conditions relevant to the orbit of AO-7.

1.1 The Spacecraft (plural):

The Delta 104 mission payload consisted of three spacecraft (at the time, another first for the Delta Program and for NASA). The primary spacecraft was ITOS-G (which upon launch became NOAA-4, the 4th in the second series of operational polar meteorological satellites. Two secondary payloads were included on this launch. INTASAT was the first of the secondary payloads included by NASA HQ on the mission. This was the first satellite for the country of Spain and it carried out an ionospheric research mission[3]. The second secondary payload was AMSAT-OSCAR-7. The 7th in a series of communications satellites, this program was initiated in 1961 by a not-for-profit group, known as Project OSCAR. The AO-7 satellite carries two linear communications transponders, 4 beacon transmitters as well as multiple/redundant digital command and telemetry systems plus one store-and-forward data experiment (a fore-runner to the AX.25 digital packet radio systems of the future). The launch of Delta 104 occurred at 09:11 PST (17:11 UTC) on November 15, 1974 from the Western Test Range, VAFB, CA. The launch and orbit achieved all mission requirements. All three of these missions were very successful and met their mission goals and objectives. INTASAT used a two-year electronic timer to assure its mission termination. It was the first of the 3 satellites to cease its emissions. NOAA-4 was retired after 1463 days of operations, fulfilling its mission on November 18, 1978. AO-7 lasted (on it's first lifetime) until mid-July 1981, outlasting both of its co-passengers. AO-7, our "sleeping beauty," is an on-going story. And, more will be written about this system in the future.

1.2 The Launch Vehicle:

Delta 104 was a standard 2310 straight-8 Thor-Delta vehicle. It employed 3 Castor II Solid Rocket Motors (SRMs); all three SRMs being jettisoned at burn-out. The first stage used the famous Rocketdyne RS-27 motor, burning LOX/RP-1. The second stage was hypergolic and used UDMH/N2O4. Thus configured, this stage was restartable. This vintage of Delta already included the DIGS (Delta Inertial Guidance System) and a Flight Computer/Sequencer. The orbit established by NOAA and NASA/GSFC, satisfied ITOS Polar Mission Requirements and was planned as a higher altitude LEO SSO[4]. The initial orbit for this set of three spacecraft is discussed below in considerable detail.

1.3 The Role of NORAD:

Established in 1858, NORAD (North American Aerospace Defense Command) was the original U.S. Dept. of Defense organization responsible for detecting, tracking, and cataloging all objects in space and for determining their orbits. The role of NORAD and its derivatives over time, is a complex story. However, certainly it is fair to say that among their many roles within the DoD, NORAD's most public facing output is the generation of the (current designation) JSpOC Space Catalog. This catalog, available on the web, can be found at https://www.space-track.org/. While NORAD's function still exists, as does the organization, within DoD, the role of maintaining the catalog of all space objects was delegated to JSpOC (Joint Space Operational Center) in May 2005. This command was reorganized in July of 2018, as CSpOC (Combined Space Operations Center) and now includes multinational elements. One must realize, given the theme of this paper, all three organizational names have changed over time, however, AO-7 has been a "space object of interest" and has been "in the catalog" during ALL of the transitions of this complex organization, with it's many roles. While taking on many new roles, the role and requirements for maintaining the space catalog has remained constant with time. The widest expression of the role of this "consistent yet ever-changing" organization is expressed via the now famous message type it produces; best known to all aerospace engineers, world-wide as **TLEs** (or two-line element sets). Each TLE set or "message" is a measurement of each object's "state vector." Put somewhat differently, into classical Physics, it contains the Keplerian element set (and associated epoch) for any given space object. From the TLEs many products can be derived. The authors realize that the raw TLEs are rarely utilized for direct analysis. Rather, they are most frequently consumed by a myriad of software applications (apps).

Curiously, our intent was to propagate the orbit *backward* in time. We wanted to know where this old satellite had been – for the last 50 years. In order to do this we found it best to "sample" the TLE sets for our object periodically into the past in a manual fashion, using only the power of an EXCEL spreadsheet and the fundamental mathematics of orbital mechanics. So far, we have not found it necessary to use the significant computational power of STK, Astrogator or even a simple forward orbit propagator (FOP) to visualize our findings. In order to demonstrate what we are about to explain, we only needed the TLE sets from this incredibly steadfast organization over about 50 years, taken one year at a time. Thanks to CSpOC and Space-Track.com we think we can explain, in some detail, a small bit of orbital mechanics none of us have likely seen before. Thank you NORAD

2.0 The Planned Orbit:

The primary payload launched by Delta 104, NOAA-4, required a sun synchronous orbit. This polar meteorological satellite's instruments were designed for a circular sun-synchronous orbit with an, arguably, high altitude for such a LEO SSO. The NOAA constellation of ITOS (Improved TIROS Operational Satellites) satellites specified an orbit as follows:

Height: $h_a = h_p = 1460$ km
Eccentricity: $e \leq 0.0015$
Semi-major Axis: $a = 7828$ km
Inclination: $101.69 \pm 0.01°$
RAAN: Ω consistent with an LTAN of 08:32

As we will see, it is the inclination (i), of this orbit, which is the most critical mission parameter. This Keplerian element establishes, along with the altitude, the degree of sun-synchronicity of the orbit. This orbit can use some explanation. In one real sense, the choice of this orbit reflects the U.S. government's lack of concern regarding space debris – at that point in history. None of the payloads on-board Delta 104 even considered the use of propulsive devices for the removal of the spacecraft from orbit. A quick analysis using SMAD[5] reveals that all three payload "objects", given their A/m values, have an orbit lifetime approaching 10,000 years. So, none of these objects are going anywhere fast for a very long time. Of primary concern to NASA and NOAA, at the time, was the ability to target the sun-synchronous inclination, since that would establish one limitation for the mission lifetime.

Many more details are know by the authors, as to how the orbit accuracy is established procedurally, using the Delta Inertial Guidance System (DIGS) in cooperation with this launch vehicle's Flight Computer. While, the process is lengthy (and beyond the scope of this treatment), the procedure involves many simulated trajectory runs utilizing "Monte Carlo" variations of key trajectory parameters (meteorological and launch vehicle performance variables are among these). The end result is the establishment of a state vector at injection (release of each payload). Most generally, this takes the form:

$$\vec{V} = (x, y, z, \dot{x}, \dot{y}, \dot{z}, t)$$

However t, in this instance, is relative to the lift-off time of the launcher, which is governed by the "launch window." And, this is always negotiated between the Delta Project Office and the customer (in this case, NOAA).

The Cartesian state vector can be re-expressed in terms of the classical Keplerian element set:

$\vec{V} = (a, e, i, \omega, \Omega, M, t)$

where:

a = Semi-Major Axis (SMA) {km}

e = Eccentricity

i = Inclination {°}

ω = Argument of Perigee (AoP) {°}

Ω = Right Ascension of Ascending Node (RAAN) {°}

M = Mean Anomaly {°}

T = Epoch [6]

One output of the statistical trajectory analysis[7] is most relevant to our long-term orbit investigations. This was the uncertainty of the inclination parameter. While the original records are no longer available to the authors, we are certain the specification on inclination was well known at the time; based on other LEO orbits from the same time frame. It can be assessed to be very close to the following values:

Mean inclination = i_{mean} = 101.69 °
(for a = 7830 km; L/V target)

Standard Deviation (1 x σ) = ±0.068°

(2σ) = ±0.095°

(3σ) = ±0.100° (99.7% probability)

These statistics are fundamental in developing the motivation for the inclination target as assessed by NASA, NOAA and MDAC (the Delta L/V contractor). And, the consequence of this trajectory analysis, performed by MDAC, is an assurance to their customer NOAA, that the inclination uncertainty of the NOAA-4 orbit would be 101.69° ± 0.100° with a 99.7% probability of occurrence. The government, at that time was always willing to "bet on" 3σ outcomes such as these at Pre-Ship Reviews.

2.0 The Achieved Orbit:

The orbit achieved by Delta 104 was remarkably good, in terms of duplicating the desired state vector. ITOS-G became NOAA-4 upon separation from the vehicle. This "injection" was the completion point for Delta for their primary mission. The details regarding the reorientation before injection are no longer known, however the reorientation would have provided NOAA-4 with a good solar pointing vector (with sun directed approximately normal to that spacecraft's 3 co-planer solar panels). After a very short coast period the Delta second stage did a re-orientation, approximately 180 from the velocity vector of the orbit and released both INTASAT and AO-7. This attitude assured good separation dynamics of the secondary spacecraft from the primary spacecraft. Prior to launch the Orbit Dynamics Group of the Delta Project at NASA/GSFC carried out an analysis of the separation dynamics of the three bodies in order to assure no re-contact would occur among the three objects. To assure that none of the payloads would re-contact with the 2nd stage of Delta, a final burn-to-depletion of the motor took place. This burn, made a small difference in inclination between the stage and the three payloads. This difference can still be observed in current TLEs. The Delta vehicle is capable of reporting it's own estimate of the orbit state vector at the time of the primary payload injection, so the first report of a nominal orbit is typically given by launch vehicle telemetry. This occurred during the time the Delta 104 vehicle was over the island of Madagascar on Orbit 0. This Keplerian version of the state vector is no longer available and is lost to time. However, this set was used by the AO-7 amateur network as input for our crude tracking software of the time, as a means of commanding the spacecraft. Most amateur radio users, however, were just listening at nominal times, broadcast before launch, with antennas pointed at their nominal azimuth on the horizon, until the AO-7 beacon signal was heard. Then each station was on it's own to figure out how to track the spacecraft.

NORAD, in 1974, didn't quite have the ability to produce TLEs for every space object every day with the "picket fence" radar. For AO-7, the first TLE set arrived on 17 November about 48 hours after launch. At the first recorded epoch, the AO-7 spacecraft had completed 20 orbits. In light of where we all are today with our personal communications devices, you might find it humorous to recall how we used to recover hightech data in 1974. There was, of course, a direct data flow and communications channel between NORAD and NASA/GSFC to deliver TLEs on all objects. They arrived via a Teletype machine, on physical *paper*. Multiple copies would emerge from the RTTY machine (yes, using carbon paper). The primary copy was immediately dispatched to the NASA STDN network and the other copies were placed in a particular set of physical boxes (sorted by object number) and were available to be reviewed by any NASA employee. AMSAT had volunteers (who were, of course, GSFC employees in the STDN network) posted in Building 12 at GSFC who would check the correct box for Object 7530 each day. The Object 7530 TLEs were then forwarded to our Radio Amateur network via HF radio and sometimes even via the AO-7 spacecraft itself, just to prove to others this could be done. The AMSAT community soon learned that orbits at an altitude of 1460 km had so little drag that a single TLE set, even when propagated forward by our simple manual methods was adequate to track AO-7 for months into the future. Hence, it was soon learned, there was no rush to deliver new TLEs around the world. Most amateurs simply used the Ascending Node crossing time and the Azimuth value at the equator crossing at that time, as their means of tracking the satellite. Simple FORTRAN programs, run on mainframes were used to do this. Then paper/cardboard "plotters" were used as a means to keep track of the spacecraft's azimuth and elevation as a function of time. The station operators (users) pointed antennas manually, in order to keep the spacecraft within their antenna beamwidths. Each operator had to also operate his radio equipment at the same time. This wasn't as difficult as it sounds because one must remember; the passes were about 25-30 minutes long, due to the high altitude of this SSO. This duration is about twice as long as for current SSO "traffic," which primarily uses orbits of 500-600 km altitude. The duration of a "modern" SSO satellite overhead pass is 10-12 minutes, by comparison. Beamwidths were correspondingly large (typically about 30°) making antenna pointing an easy affair.

5.0 Key Events of AO-7 Spacecraft During the Past 50 Years - Why is a 50 Year Mission Interesting?

While the details of the "life" of AO-7 are discussed in another paper published in the Proceedings[8] we are interested here in what caused this particular investigation. The lifetime of AO-7, our "Sleeping Beauty" spacecraft can be characterized by three distinct phases: 1) the primary lifetime of the mission (from November of 1974 until July of 1981), 2) the "sleeping" phase of the mission caused by the fail-short of *all* of the NiCd battery cells (from July 1981 until, July 2002 and 3) the extended mission of AO-7, enabled by a second fail-open of (at least) one of the shorted NiCd cells. This we call the SECOND LIFE of AO-7. This "second failure" now allows the spacecraft to operate "normally" but, only during sun-lit portions of the orbit. And this 3rd phase of the operation of AO-7 continues to this day.

One can understand our lack of enthusiasm during the second period or sleeping phase of AO-7's "lifetime." There was little interest even among the authors in keeping track of the orbit of a deceased spacecraft – life moves on. However, we wish to emphatically note here that NORAD's job isn't over when a spacecraft becomes inoperative in space. The TLE's continue to be published unabated. ***This*** mission is perhaps most important because *it reminds us continuously* of this matter. *Orbit debris is dangerous!*

There was a renewed enthusiasm, to be sure, when AO-7 entered it's third phase and for one particular reason, among others. The spacecraft is now functional only during sunlight and Appendix 1, Figure AP1-2 demonstrates that we had anticipated the satellite orbit to be drifting toward a "twilight" (06:00 LTDN-18:00 LTAN) orbit. Twilight SSOs have the unique feature that spacecraft in such orbits never go into eclipse. And, such an orbit is ideal if your spacecraft *no longer has a battery*! Users of the system had noticed that the satellite was, during some times of each year, in continuous sun. When a spacecraft drifting in LTDN, reaches a mean sun time of approximately LTDN=7:30, at this higher altitude, the changes in mean sun time over the year, caused by the Earth's eccentricity (as explained in Appendix 1) cause the satellite to "wobble" into and out of the 100% sun condition. AO-7 users had for several years, been observing this condition. So, it became of increasing interest to determine what was going to happen to the orbit of this old spacecraft, in order to plan it's future utilization more effectively. However, since other projects in life take higher priority than old amateur radio satellites (sadly), we didn't get around to this task until last year. And, what we discovered surprised us.

6.0 *Surprise! – AO-7's Real Orbit Propagation:*

It is hardly surprising, since our first interest was in sun-to-orbit plane angle (Φ) over time, that our primary interest in this long-term data set was in the RAAN (Ω) changes of the orbit. However, as this process was thought to be driven entirely by the Earth's J_2 Perturbation, we also wanted to observe changes in **a** or **n** (due to drag) and any changes in inclination (which we thought would be very small) and the eccentricity (**e**) of the orbit, which were small to begin with, we expected the time-rate changes might be even smaller. These three orbital elements affect the orbit J_2 Perturbation as per Appendix AP-1, equation (5). We started our investigations using Space-Track.org, where one can find every TLE set for every space object in Earth orbit (except the classified ones) over the last 60 or more years. Since we were not interested in observing the "wobble" in mean sun time caused by the eccentricity of the Earth's orbit *about the sun*, and since we wanted to make our calculations as simple as possible, we used an old astronomy trick. We sampled the Ω parameter of the orbit by grabbing TLEs on March 21 of each year. Why? On Julian Day 80.0000 (March 21 at Noon UTC) the Sun, as seen from the Earth crosses the line between the center of the Earth and the spot in the sky known as "the first point of Aires" (♈). This is also the *place and time* where the projection of the Earth's equatorial plane intersects the ecliptic plane. And, this is also the definition of the epoch for *vernal equinox*. Figure 1 demonstrates the conditions given in Figure AP1-1 at Vernal Equinox. Φ = RAAN (Ω) at that epoch.

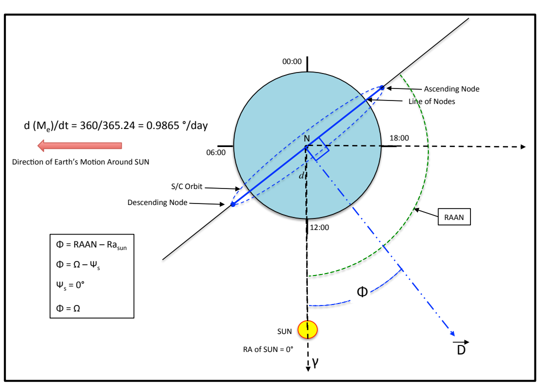

Figure 1: Simplified Geometry as Seen from the North (On March 21 @ 12:00 UTC)

To say that we were surprised by these results of our findings, as shown in Appendix AP-1, is an understatement. To be expecting a linear translation of RAAN over time and to find a sinusoid was virtually breathtaking. We also anticipated very little change over time in the inclination and we certainly did not expect a sinusoidal behavior in that orbital element as well. Let's try to put this into perspective. This is NORAD data, not data computed by the authors. We only added a small correction in RAAN to bring the TLE value in line with an 80.0000 day-of-year Epoch - at Vernal Equinox. We expected to demonstrate a plot similar or identical to Appendix 1, Figure AP-12 for RAAN.

7.0 *What's Been Going on with AO-7's Orbit?*:

When we finally got our heads around the fact that this must be something we hadn't thought about, and that this data was real…we finally got back to business. We can make the following immediate observations about the RAAN and inclination of AO-7's orbit over this nearly 50-year period (as you can observer yourself in Appendix 1, Figures 4 and 5):

1) The period of both sinusoidal oscillations is approximately 29 years

2) The amplitude in RAAN change is approximately 80° (±40°).

3) The amplitude in Inclination change is approximately 0.6° (±0.3°).

3) Starting from the first TLE, the RAAN function is a Cosine. And, the Inclination function is a -Sine function. So, the derivative of the RAAN is proportional to the inclination.

4) The cosine function associated with the RAAN starts at (or very near) injection time. The negative trend in the inclination (-sin) also starts very nearly at injection.

5) The center (mean value) of the RAAN cosine function is approximately at 88+ degrees (or very nearly at 90°), which is the condition for a "twilight" orbit). In other words, the oscillation seems to center on the twilight orbit condition.

6) The center (mean value) of the inclination – sine function seem to be very close to the value for i_{sso} or 101.69°.

7) The AO-7 orbit seems to have, on average, remained sun-synchronous but, *oscillates* around a mean LTAN/LDTN of 6:00 – 18:00 or $\Omega = 90°$ $= \Phi$ at Vernal Equinox.

7.0 *The Origin of AO-7's Orbit Perturbation*
In Appendix 2 we explore in more detail the physics of this apparent perturbation acting on the inclination of AO-7's orbit, which, in turn, is causing a change in AO-7's RAAN value. It is clear from simple observation that this perturbation is not being caused by the Earth J2 term. To first order the J2 perturbation, is linear, not sinusoidal. We found that, once simplified, the equations of motion can be solved when represented as a second order linear differential equation. The solution, involving various simplifying assumptions, is provided in Appendix 2. We find from the results of this evaluation:

1. There is a strong coupling between the Solar-induced torques and the inclination of the S/C orbit, as a function of time. Changes in the inclination couple via the J2 perturbation and affect the RAAN vs. time.
2. The equations of motion are identical to those observed for a simple harmonic oscillator
3. The derived scenario we completed, when applying appropriate astronomical constants, reasonably closely fits the observations given above, in terms of the amplitude and period of the oscillation data observed in the CSpOC TLE data, which we processed.

8.0 *Discovery of the Solar Gravity Gradient Force Acting on SSOs:*

As is always the case when circumstances like this occur, it was important for the authors to know if this was the first observation and analysis demonstrating this phenomenon. It is essential to note that, AFTER, we completed the basic analysis and derivations associated with our findings we did a fairly exhaustive search of the literature to determine if this phenomenon had been previously reported. Each time we did a search, we used different Internet search criteria and on the 3rd attempt we identified a paper reporting the same type of derivation[9]. This paper, written by a NASA physicist, K. Duck, was published in a limited-publication, NASA X-document format back in 1975. We believe, now fully, that his development, while carried out very differently, identifies the same Solar-GG effect we first observed at the end of last year. We believe our discovery to be a fully, blind, re-discovery.

We can therefore, not claim to be the first individuals to validate the source of this perturbation. However, we do believe that we are the first to report the observation of this phenomenon and certainly by demonstrating the observation of over 1.7 cycles of its 29-year RAAN cycle periodicity.

The ironies of the story of this discovery in relationship to the genesis of the AO-7 spacecraft have a "believe it or not" quality to them. All below is true and happened (did occur):

1) Neither of the authors knew nor has ever met Ken Duck.

2) Ken would have worked at NASA/GSFC in 1974, when he was developing his ideas about the Solar-GG torques acting on SSOs. He most likely worked in Building 6 on the NASA/GSFC campus.

3) One of the authors, Jan King worked in Building 22 at NASA/GSFC in 1974 and was competing the construction and testing of the AO-7 spacecraft over in Building 7.

4) AO-7 completed environmental testing in mid-1974, while K. Duck would have been completing his analysis. K. Duck and J. King would have been working less than 300 meters from one another while this all transpired.

5) Of all of the possible orbits K. Duck could have used to validate his theory, he decided to use the NOAA-4 (ITOS Series) Orbit to validate his Solar-GG theory. That is precisely the orbit into which AO-7 was launched.

6) K. Duck will have published his X-document within a few months of the launch of Delta 104 (and AO-7). Neither of the other two payloads on that mission lived long enough to identify the Solar-GG perturbation, while they were still functional. It would have been possible to use CSpOC TLEs, however, to find this condition for those objects, had there been a motivation to do so, just as we have done.

7) 50 Years after K. Duck's publication, AO-7, occupying the same orbit as he had imagined, finally validated his very long period predictions and proved it is all TRUE.

We've shown you nearly two cycles of the RAAN and inclination oscillations, each with a period of 29 years. We've demonstrated an independent proof of this coupled, restricted condition, 3-body problem. It is a physics case where the sun works in conjunction with the Earth to modify the orbit of another body in orbit around the Earth. The spacecraft that occupies the orbit under consideration happens to still be working. That is the only reason you are reading about this now.

We don't think Aerospace Engineering gets too much more interesting than this story has been to us. We hope you agree.

7.0 Acknowledgements:

While the authors did most of the work for this paper independently, while cooperating only with one another, we most sincerely wish to thank North American Air Defense [now the Combined Space Operations Center (CSpOC)] for making this data available for free, but, more importantly, for their tireless effort to keep all space objects safe. Not only does CSpOC track every single object - now with incredible accuracy - they have been doing this for 60 years! And, what is mind-boggling is the continuity of the whole operation. We can see from this small project, reported here, that there were good years and bad years for NORAD back in the 1970's and 1980's. And, there is clear evidence of "picket fence" upgrades in our data, however, there was never a time in 50 years of data taking that the system was shut down for any duration that prevented this wonderful organization from "doing it's job." One can't quite get one's head around the number of objects launched multiplied by the number of orbits per object completed, whether these objects are big-or-small (10 cm small), alive-or-dead, they are targeted, tracked and reported, no matter what. It doesn't get easier as the number-of-objects in space are now increasing exponentially. Yes, we all know this process is getting more automated. But, one cannot NOT help but think, occasionally, about the dedication that goes into this process, by many people, every day - always. Thank you, thank you, CSpOC.

9.0 References:

1. King, Jan A. et al., *AMSAT-OSCAR-7: A Still Operational, Small Satellite History Lesson*, SSC24-I-01, Proceedings of the 38th Annual AIAA/USU Small Satellite Conference, August 5, 2024.
2. Ibid,1., p. 2, The NiCd battery initially failed in a mode such that all cells failed SHORT. Then the battery failed again, with at least one cell failing OPEN. This 2nd failure returned the spacecraft to service but, only when it is NOT in eclipse.
3. INTASAT conducted electron density and scale height measurements using coherent beacons in the 40-42 MHz frequency band. This research program was carried out under the auspices of the Comision Nacional de Investigacion del Espacio (CONIE), Spain's National Commission for Space Research.
4. It is important to note that this circular orbit's altitude, at 1460 km is very high by today's standards. The secondary payloads were not required to de-orbit. Debris mitigation had not yet been considered for small satellites. Indeed the notion of small satellites didn't yet exist. AIAA/USU did not exist.
5. Wertz, J.R. et al, *Space Mission Engineering: The New SMAD*, Space Technology Library, Micorocosm Press, Hawthorne, CA, p.1032, (2011).
6. For expression of TLE's CSpOC utilizes the Julian date/time, where 12:00 UTC represents the start of the Julian day. We will use this reference to time in this analysis.
7. This trajectory analysis was codified within the McDonnell-Douglas system in a document that was known at the time of Delta 104 as the Detailed Test Objectives (DTO). This document was the ultimate representation of the trajectory agreed upon between the Delta Project Office and the customer for any given mission. The DTO included, as it's primary component, a trajectory "plus time run" averaged over many trajectory simulations, each containing randomly selected changes to variables, complying with the

statistics of each parameter. This is classically known as a “Monte Carlo” process. The inclination as a component of the state vector at spacecraft injection is one of the outcomes of this statistical analysis.
8. Ibid, 1., p.26.
9. Duck, Kenneth, L., *Long Period Nodal Motion of Sun Synchronous Orbits, ntrs.nasa.gov,* NASA/GSFC, Doc N76-10175, 1975.

APPENDIX 1: The Mission Orbital Observations and Analysis Employing CSpOC TLEs and Other Measurement Methods

1.0 Orbital Elements:
The Keplerian representation of the state vector, as described in the main body of the paper, will be used in our analysis in order to be consistent with the TLE's, with one exception. NORAD (CSpOC) employs an orbital mechanics system known as *Brower Mean*. This system uses an alternative parameter to the SMA (a) known as the "mean motion." One expression of the mean motion is the number of orbits (revolutions), which occur per day, for the object being assessed.

The mean motion **n**, can be exchanged for the classical Keplerian element **a**, the semi-major axis. To use this particular form of the state vector, we show here the simplified, defining relationship:

$$n = \sqrt{\frac{\mu}{a^3}} \qquad (1)$$

where:

$\mu = G * m_e$ = gravitational constant for the Earth

a = spacecraft orbit semi-major axis

In practice, we use a more detailed version of this equation in order to include the J_2 perturbation of the Earth. The key thing to note at this point is that we adopt the form of orbital elements used by CSpOC rather than the classical Keplerian elements, although they only differ by one element. Fundamentally, we have replaced **a** by **n** in the element set. The most important thing to keep track of here; if we think about drag as decreasing the semi-major axis (**a**) of the orbit, we must think, in CSpOC terms, that we are instead, increasing, marginally, the number of orbits completed by the spacecraft per unit of time. We will elaborate slightly below.

The first delivered TLE set, still available to anyone who uses Space-Track.org yields the following outcome for Object 7530:

Julian Epoch: 74321.27777050

n = 12.5309952 rev/day

e = 0.0013024

i = 101.7379°

ω = 213.6762°

Ω = 5.9721°

M = 146.3438°

Orbit# = 20

2.0 The Orbit Injection Error in Inclination:
From this TLE set the first task is to find the semi-major axis value, given the mean motion as per the first TLE set. Our aim is to determine the error in inclination at injection and find out how fast the real orbit is precessing (in terms of dΩ/dt). As a means of reducing the scope of this paper it is appropriate to cite two sources, which present the important Earth J_2 perturbations as they apply to SSO orbits[1,2]. We have utilized equations in the references here to compute the sun synchronous inclination, given the semi-major axis, derived from the mean motion (**n**), the inclination and, as a secondary player, the eccentricity of the orbit, all at an epoch as close to injection as feasible. We cite four strategic equations from the above sources in order to emphasize their role in determining sun synchronism for the AO-7 orbit. These equations framed our thinking regarding how the orbit would propagate into the future:

$$d\Omega/dt = d\,(M_e)/dt \qquad (2)$$

This is the fundamental statement of sun synchronism; it locks the first derivative of the RAAN of the spacecraft's orbit to the first derivative of the Mean Anomaly (the rate of motion) of the Earth in it's orbit about the Sun.

The J_2 perturbation of an Earth-orbiting satellite in Right Ascension is:

$$\dot{\Omega} = -\frac{3}{2}\ J_2\left(\frac{R_e}{a(1-e^2)}\right)^2 \sqrt{\frac{\mu_e}{a^3}} * \cos i \qquad (3a)$$

or, alternatively,

$$\dot{\Omega} = -\frac{3}{2}J_2\left(\frac{1}{(1-e^2)}\right)^2 n * \cos i \qquad (3b)$$

Equation (3a) uses the classic Keplerian orbital element **a**, while (3b) assumes the use of mean motion, as has been employed by CSpOC in the TLEs.

(2) and (3a) can be combined and re-written to yield an equation suitable for finding the inclination required for sun-synchronism, given a, e and observing that $d(M_e)/dt$ for the Earth's orbit = 360.00/365.24 (deg/day).

$$i_{sso} = cos^{-1}\left(-\frac{2\dot{\Omega}}{3*J_2\left(\frac{R_e}{a(1-e^2)}\right)^2\sqrt{\frac{\mu_e}{a^3}}}\right) \qquad (4)$$

Where, in the above equations $\mu_e = Gm_e$; the gravitational constant for Earth. Finally, we use an engineering version of the relationship between semi-major axis and mean motion, which assisted us in moving between the two orbital systems:

$$n = \frac{8681660.4}{a^{3/2}}\left(\frac{1+1.5J_2\left(\frac{R_e}{a}\right)^2}{(1-e^2)^{1.5}}\right)\left(1-\frac{3}{2}\sin i\right)^2 \qquad (5)$$

From the above we can calculate that the SMA (a) as determined by (5), if we know n, and the sun synchronous inclination, as determined by (4). The SMA result is:

a = 7827.2290 km

And using this value in (4), we find:

i_{sso} = 101.6859°

And, the inclination of AO-7 as reported by NORAD in the first available TLE set, as given above, is:

$i_{AO-7} = 101.7379°$

Thus, we note that the launch vehicle error at injection (assuming there has been very little change in a, i or e during the first 20 orbits) was approximately:

$\Delta i = i_{AO-7} - i_{sso} = \mathbf{+0.0502°}$ (6)

3.0 What was Anticipated? The Expected Propagation of the Orbit of AO-7:
We note the error in inclination of the achieved orbit at injection, is slightly higher than the inclination for the sun-synchronous rate. The J_2 perturbation is given in (4) above. This classic equation yields a constant rate of change in the rotation of the plane of the satellites orbit w.r.t. say, the equator of the Earth. One could state that the nodes of the orbit are precessing at this rate. This rate will remain constant so long as (using the NORAD system) **n**, **e** and **i** remain constant. We anticipated no variations in the eccentricity of the orbit because there are virtually no perturbing forces, given the orbit is very nearly circular to begin with and the orbit altitude is very high, implying differential drag should have very little effect. The **a** of this, or any, orbit is subject to atmospheric drag. So, if **a** decreases, we would anticipate **n** to increase, as noted above. However, given the orbital altitude, this orbit has very low drag. As noted previously, the orbit lifetime expectation for all of the objects associated with Delta 104 is approximated to be about 10^4 years. Therefore, the decrease in orbit altitude over 50 years and the increase in mean motion over that same period would be expected to be quite small. We did want to verify the changes in n so as to observe the nature of the anticipated small increase in mean motion due to drag. This was carried out and details are given below. Inclination was, once again, one of the most interesting parameters, subject to long-term variations. We had noted small variations in inclination with other Amateur Radio satellites, most notably AMSAT-OSCAR-6. This was based on NORAD TLEs. However, we couldn't particularly attribute these observed changes to a secular variation in the eccentricity. We also were aware that a variation in inclination would have a direct effect on changes in the RAAN progression via the J_2 perturbation. We were, therefore keen to observe the changes in **i** and **Ω**, given the availability of nearly 50 years worth of TLE data.

We return to the injection conditions of the AO-7 orbit and note again the injection error in inclination given in (6). We look at the consequences of this error in a slightly different manner by using equation (5) to compute the drift rate for AO-7's actual achieved orbit vs. a notionally perfect SSO using the J_2 perturbation as literally the driving force for obtaining sun synchronism.

$$\dot{\Omega} = 0.989636 \frac{deg.}{day} = 361.4546 \frac{deg.}{year}$$

$$\dot{\Omega}_{SSO} = 0.985626 \frac{deg.}{day} = 360.0000 \frac{deg.}{year}$$

And, then, we observe that the error in inclination at injection will result in:

$$\Delta\dot{\Omega}_{AO7-SSO} = 0.004010 \frac{deg.}{day} = 1.464612 \frac{deg.}{year} \qquad (7)$$

We conclude, based *only* on the J_2 perturbation of the RAAN, that in 50 years (or, on 15 November 2024) the AO-7 orbit will have advance by:

1.464612 deg./year X 50 years = 73.2306 deg.

The AO-7 satellite was expected to be ahead of the ideal SSO by this amount in RAAN by that time. This can be converted to mean sun time for the orbit. We note, unlike a majority of SSOs, this orbit was defined not by it's ascending, but rather, by it's descending node. These terms are equivalent. They are simply separated by 12 hours in time or 180 degrees in Right Ascension. We believe NOAA specified an 08:32 AM LTDN. Our recollection of the rationale for that choice is that a morning descending node over North America (NOAA-4's primary sensor coverage region) was accomplished more effectively by the angle of the orbit plane w.r.t. that coverage area. What the first TLE's tell us about both AO-7 and NOAA-4 is that the LTDN achieved can be determined by observing the relationships between Ω, the Earth-Sun line and the *First Point of Aires* (denoted as γ). This is the direction (within the Constellation Aires) designated as 0.0000° for the RAAN. Figure AP1-1 depicts this relationship. Using the Julian epoch of 80.0000 day as the zero angular reference for γ (first point of Aires) we can write the equation for the mean right ascension (RA) of the sun as:

$$\overline{RA_{SUN}} = \frac{dM_e}{dt}\left((Julian\ TLE\ Epoch) - Epoch\ at\ Vernal\ Equinox\right)$$

Which for our situation is:

$$\overline{RA_{SUN}} = \frac{360}{365.24}(JTE - 80.000) \qquad (8)$$

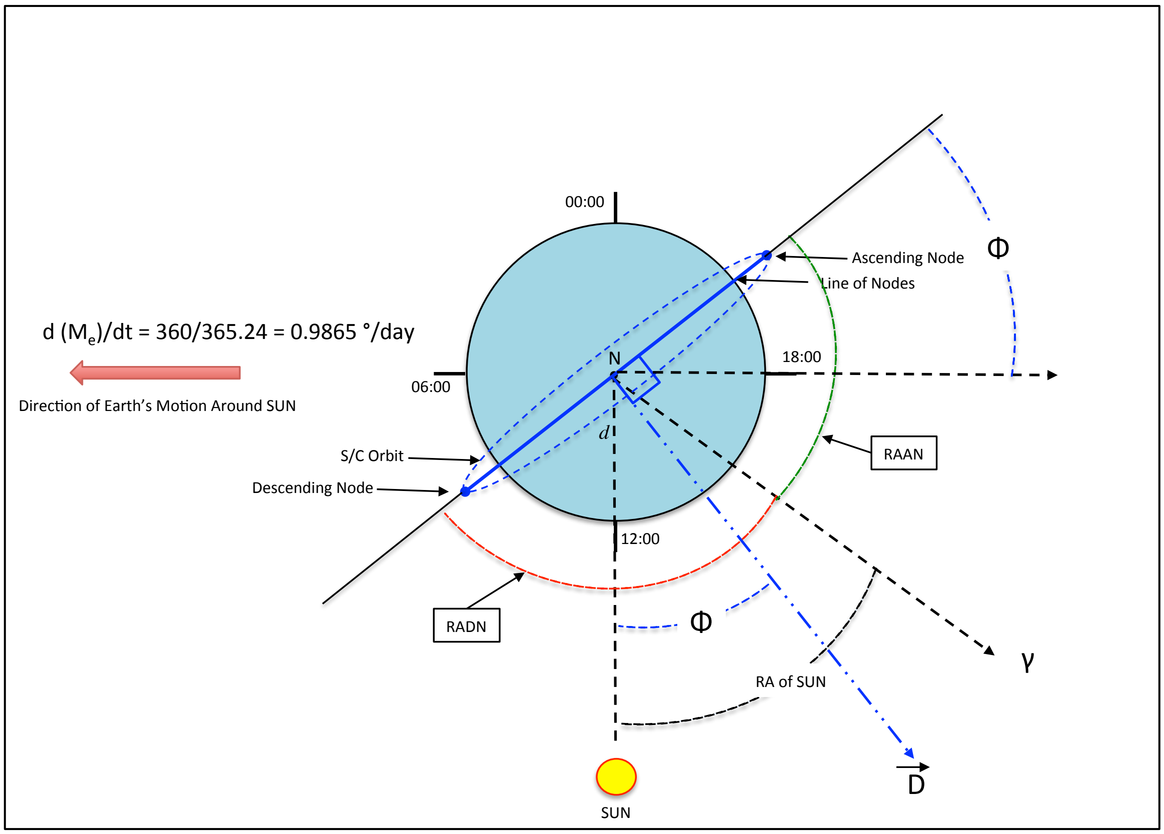

Figure AP1-1: Orbit Plane, Sun Angle & RAAN/RADN

And, by inspection of Figure AP1-1 we note:

$$Sun\ Angle = \Phi = RAAN - \overline{RA_{SUN}} \qquad (9)$$

We utilize the first AO-7 TLE set; extracting the Epoch; and with equation (7) we note:

$$\overline{RA_{SUN}} = \frac{360.00}{365.24}(321.2778 - 80.0000)$$

And, then continuing:

$$\overline{RA_{SUN}} = (0.98561) * (241.2778) = \mathbf{238.0229°}$$

We then can use (8) to determine the sun angle on the AO-7 orbit plane:

$$\Phi = RAAN - 238.0229°$$

Next we use the RAAN (Ω) value from the first AO-7 TLE set:

RAAN = Ω = 5.9721° = 365.9721°

RADN = 180° – Ω = 180° - 5.9721° = 174.0279°

Φ = (180°- RAAN) - RA_{SUN} = (180° - 174.0279°) – 238.0229° = 5.9721° - 238.0229 °

Or, alternatively:

Φ = 365.9721° - 238.0229°

Φ = 127.9492°

Converting this to a mean sun time:

LTDN = (127.9492°/15.0000) = 8.52995 = **08:31:47.81**

And, since LTAN = LTDN + 12 Hours:

LTAN **= 20:31:47:81**

We can see then, that the orbit injection state vector seems extremely close to nominal conditions, so far as inclination is concerned and, the mean sun time of the orbit (the LTDN) is only off by about 12 seconds from the 20:32:00 local time specification. This doesn't count for the propagation error of 20 orbits between injection and the first TLE set, which we take to be very small due to the nearly perfect sun-synchronism attained. So, Delta-104 certainly worked as it was intended to.

Given the orbit at injection, while quite small, there is an error in inclination as described above. The anticipated effect of this error, as the sign of the error was "+", is for the orbit to advance faster than sun-synchronism, as calculated above, by 1.46 deg. per year. And we anticipated that the J2 perturbation would be dominant. We, therefore anticipated, in the past 50 years the orbit would move ahead in mean time by 73.2306 deg. or 4.88204 Hours.

This would give a LTDN time after 50 years of:

$$LTDN_{50\ years} = LTDN_{injection} + \dot{\Omega}_{AO7-SSO} * 365.24\ \frac{days}{year} * 50 =$$

08:31:48 + 4:52:55 = **13:24:43** local time

We anticipated a linear change in mean sun time, over time, due to the error in inclination at injection; dominated by the J_2 perturbation of the Earth. We see this represented in Figure AP1-2. Here we show the anticipated change in sun angle, over time, as opposed to local time change.

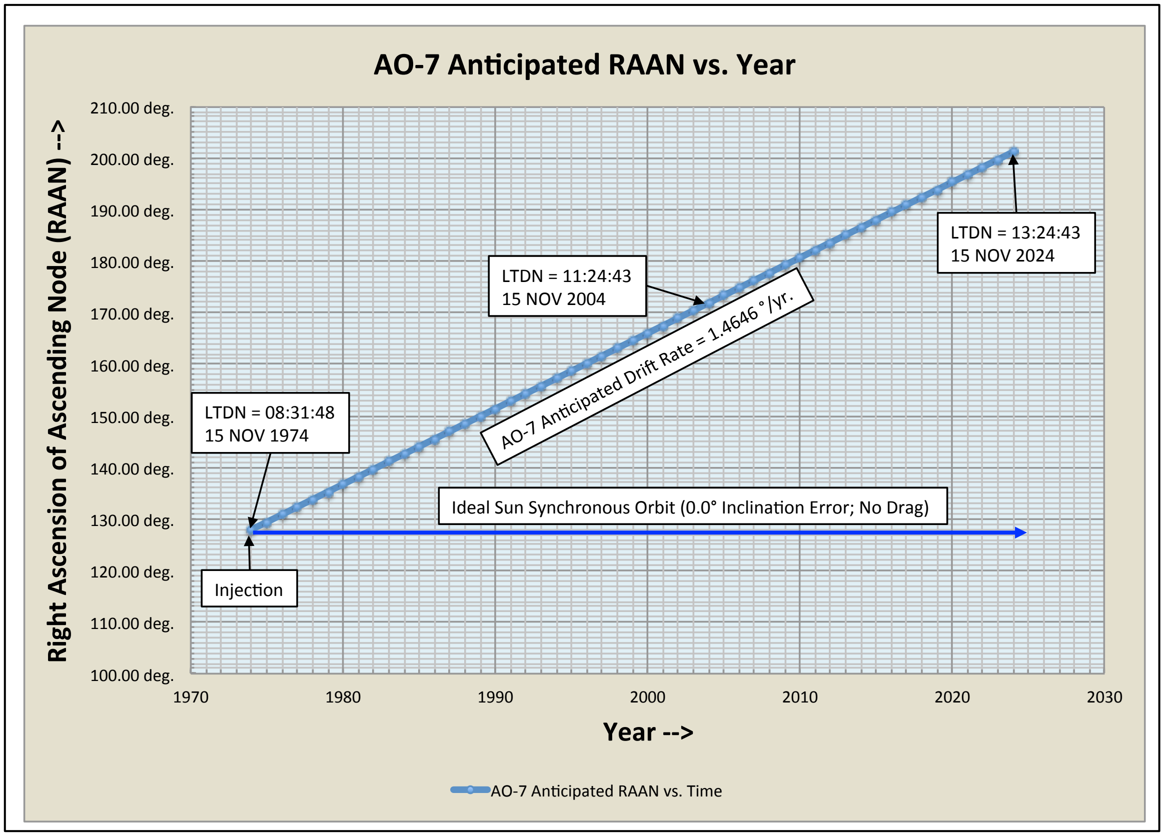

Figure AP1-2: Anticipated Drift in AO-7's RAAN Due to Earth's J2 Perturbation

In this section, for completeness, we also want to identify another orbital mechanics phenomenon. This occurs when observing the details of sun-synchronous orbits. The differential equation (2) is the definition of sun-synchronism. It locks the J_2 Perturbation of a satellite's Earth Orbit to the Mean Anomaly (M_e) of Earth's orbit around the Sun. So far, we've been "fiddling with" terms on the left-hand side of this rate equation. However, so far, we've simply used the *mean anomaly* of the Earth's orbit, not the *true anomaly*. We have assumed $d(M_e)/dt$ is a constant: the Earth moves around the Sun by an angle of 0.9856 deg/day. However, most of us are familiar with the classic Analemma in Declination and Time of the Earth's orbit.[3] This particular figure, present on most commercial globes of the Earth, simply observes that the Earth's orbit is not circular but, has an eccentricity of approx. 0.0034°. The equator is also tilted by 23.5° w.r.t. the ecliptic plane, however, this second factor can be ignored in this particular discussion. The net effect of the eccentricity is a lead/lag motion in sun angle relative to the satellite orbit plane, which imposes itself on the nominal sun-synchronicity of an SSO. This is demonstrated in Figure AP1-3. All SSOs exhibit this behavior, with a maximum lead in mean sun-angle (Φ) relative to the mean value in November and a maximum lag in Φ during the month of July of each year. The amplitude of the variation in Φ is about ±4.85°. While this factor is frequently taken into consideration in spacecraft design it is a well-known effect and we observe that it enters into the sun-synchronous relationship by means of the true anomaly of the Earth's orbit, thus, via the right-hand side of (2). We will not address this phenomenon further here. As you will soon see, this variation represents a kind of "noise" relative to our observations of AO-7's longer-term sun-synchronous behavior.

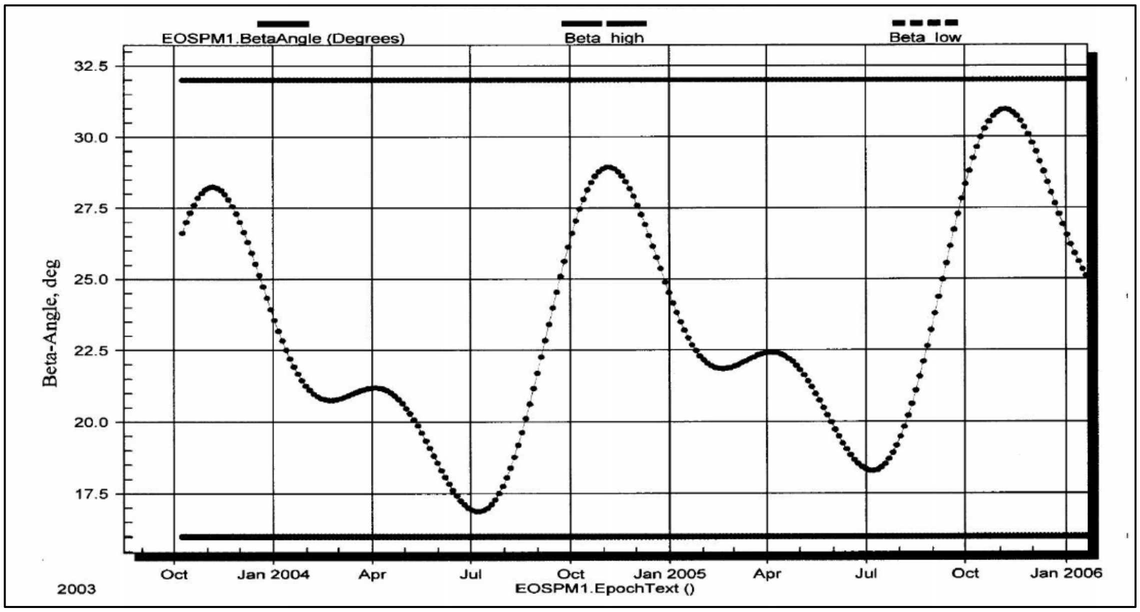

Figure AP1-3: Observed Annual Variation in Sun Angle of a Typical SSO

4.0 Using TLEs to Determine the AO-7 Orbit Behavior:

We chose to sample our TLEs using the Julian Date of 80.0000 days for each year during the orbit lifetime of AO-7. We note again for completeness, the Julian day starts at 12:00 (noon) UTC. Now, the TLE's don't always occur for our object (7530, 74089B) at exactly 80.0000. But, every year, NORAD got around to us as soon as they could. And in some years we observed the time delay in sampling our object, were off by as much as 25 days from γ=0.0°. Thus, an adjustment in the true Ω was in order. Since the spacecraft and it's orbit advance in angular rate by an amount of the J_2 Perturbation = $\dot{\Omega}$, and this is also close to 0.9856 deg./day, we can adjust the RAAN from the TLEs to bring it back to the it's value at Epoch 80.0000. We also, chose to capture the inclination value for the orbit from the same TLE from which we captured the RAAN value.

So, we sampled the NORAD TLEs for each year at the epoch closest to 80.0000 and recorded the RAAN and inclination from that TLE set. As noted here, we manually corrected the TLE epoch error in RAAN to represent the appropriate RAAN value at the epoch 80.0000. Figures AP1-4 and AP1-5 show the results. Nothing more than an Excel spreadsheet has been employed to process the data shown.

5.0 Using TLEs to Determine the Change Behavior of Other Orbital Elements:

We also want to look into the variations in mean motion and eccentricity of the same orbit, to see if other surprises might be hiding in the CSpOC data. We next present the long-term variations in mean motion. As drag acts on all LEO orbits, we expected to see an increase in **n** (in revs/day), which is consistent with a decrease in semi-major axis as the drag works on the perigee altitude to reduce the apogee altitude, first. Figure AP1-6 shows us an increase in **n**, at a nominal rate of 72.5 ppm revs/orbit/year. This was about as expected. However, an interesting small variation with about a 9-10 year period shows up in this data. Further adventures in orbital mechanics seem to present themselves here. However, we leave this one to future investigations.

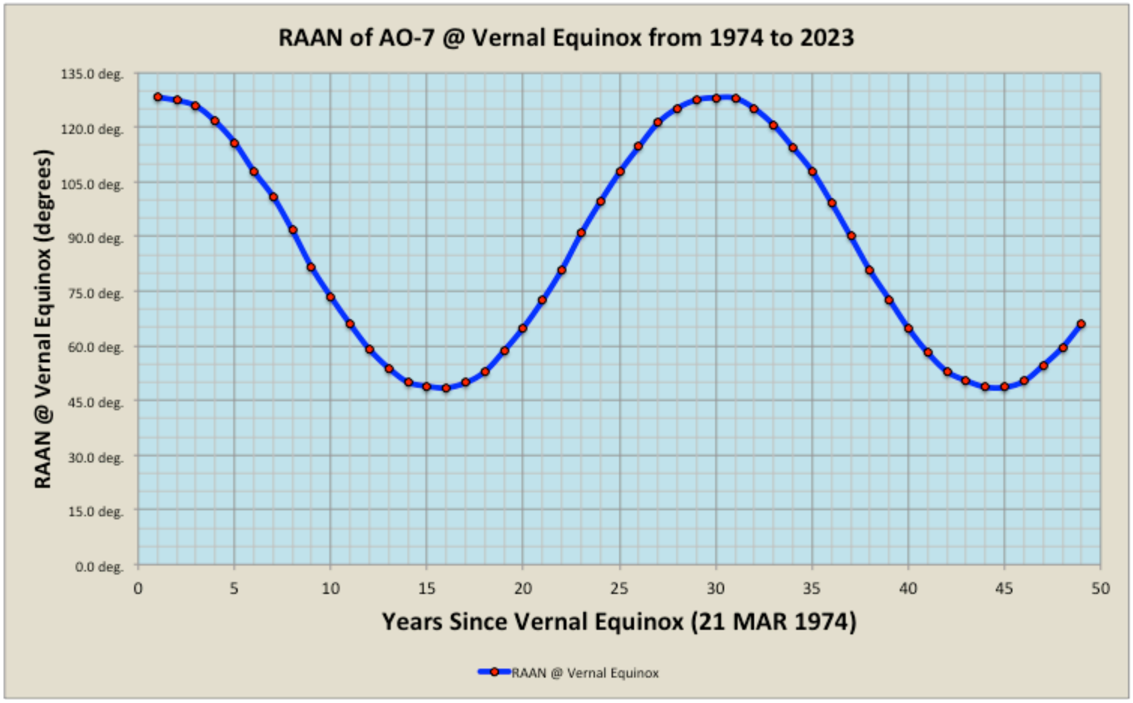

Figure AP1-4: RAAN of AO-7 Orbit at Vernal Equinox Each Year

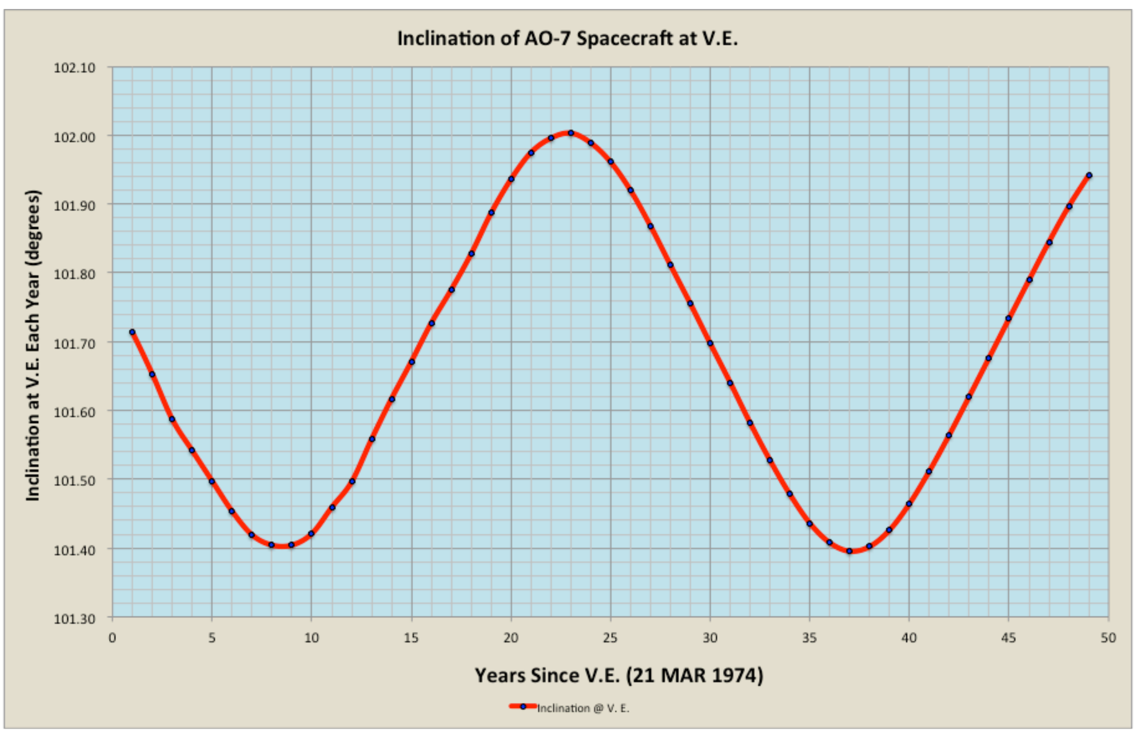

Figure AP1-5: Inclination of AO-7 Orbit at Vernal Equinox Each Year

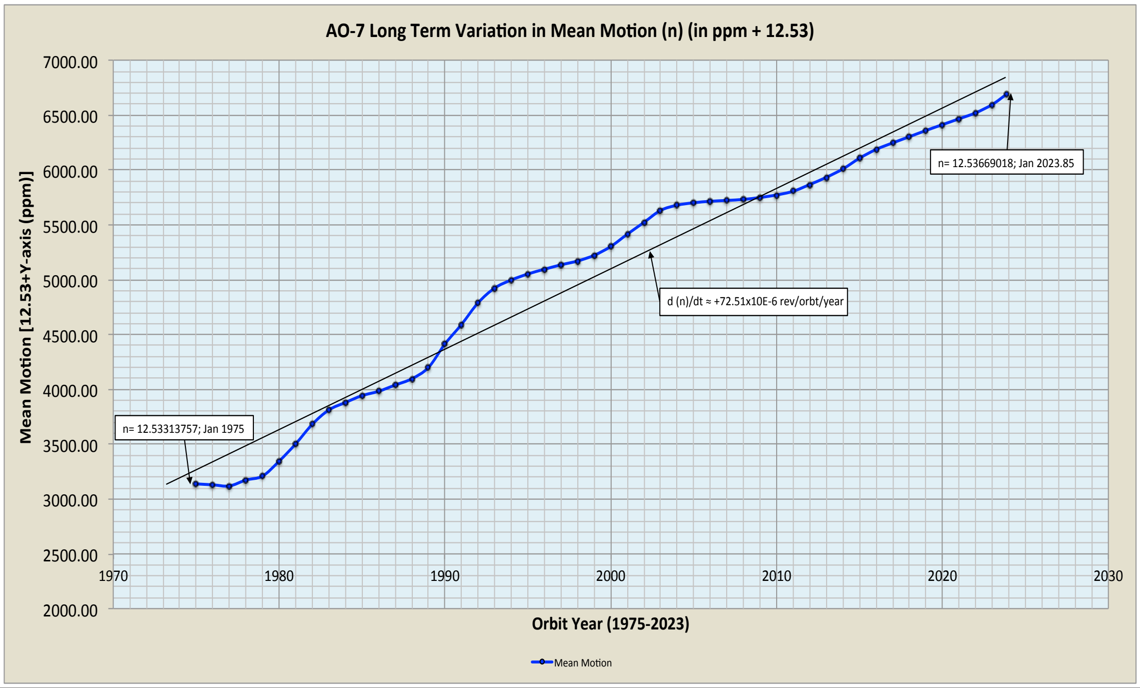

Figure AP1-6: Long-Term Variations in Mean Motion; AO-7 Orbit; 1974-2023

Investigations into the long-term variations in eccentricity were also carried out using the same method of 1 year sampling of that orbital element. Figure AP1-7 yields a little more interesting information. We strongly suspect that the change in the character of the data at about year 28 after launch is more likely due to an adjustment to the NORAD Picket Fence Radar (BPF properties?) than it is due to orbital mechanics of any sort. We cannot, however, explain the apparent damped "ringing" effect we see earlier in the mission lifetime (which has a frequency of perhaps 4-5 years in duration). Our curiosity abounds. Other shorter-term investigations were conducted using the CSpOC TLE database. These are also subjects for another day.

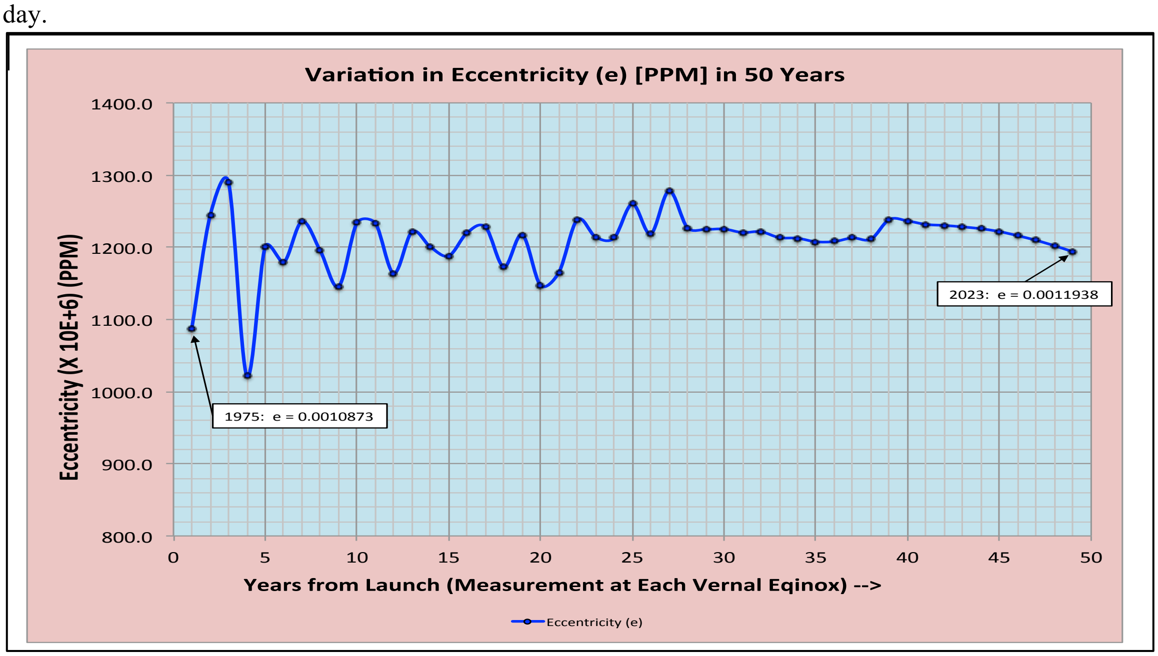

Figure AP1-7: Long-Term Variations in Eccentricity; AO-7 Orbit; 1974-2023

6.0 Using TLEs to Investigate the Behavior of the Other Delta 104 Payloads:
We return to our primary observed change-from-expectation: the clear coupling between the inclination of the AO-7 orbit and its RAAN. Initial thoughts on the source of, what could only be called at that point, a significant, apparent; orbit perturbation included non-terrestrial bodies and variations in solar pressure acting on the body of the spacecraft under varying conditions. We then recalled that CSpOc has also reported the TLEs for our two co-passengers (INTASAT and ITOS-G/NOAA-4) as well as the 2nd stage of Delta-104. The two other spacecraft, although long-since silent, are in virtually identical orbits, different in Keplerian elements primarily because of the angular difference in their separation vectors and their separation spring velocities at the time of their injection into orbit. The latter generated speed variations between the bodies of less than 3 meters/sec in ΔV. Since INTASAT's surface-area-to-mass ratio (A/m), and shape factor were very similar to AO-7 we decided to investigate the orbit of NOAA-4 instead. It has a somewhat large A/m as compared with AO-7 and, when properly oriented during its lifetime - because of it's solar arrays - it had a projected area of about 3.5 square meters, while AO-7's projected area was only about 0.2 m^2. This would cause solar pressure to be significantly higher for the primary payload. If solar pressure had played any role in the orbit modifications, it should show up in the orbit perturbations.

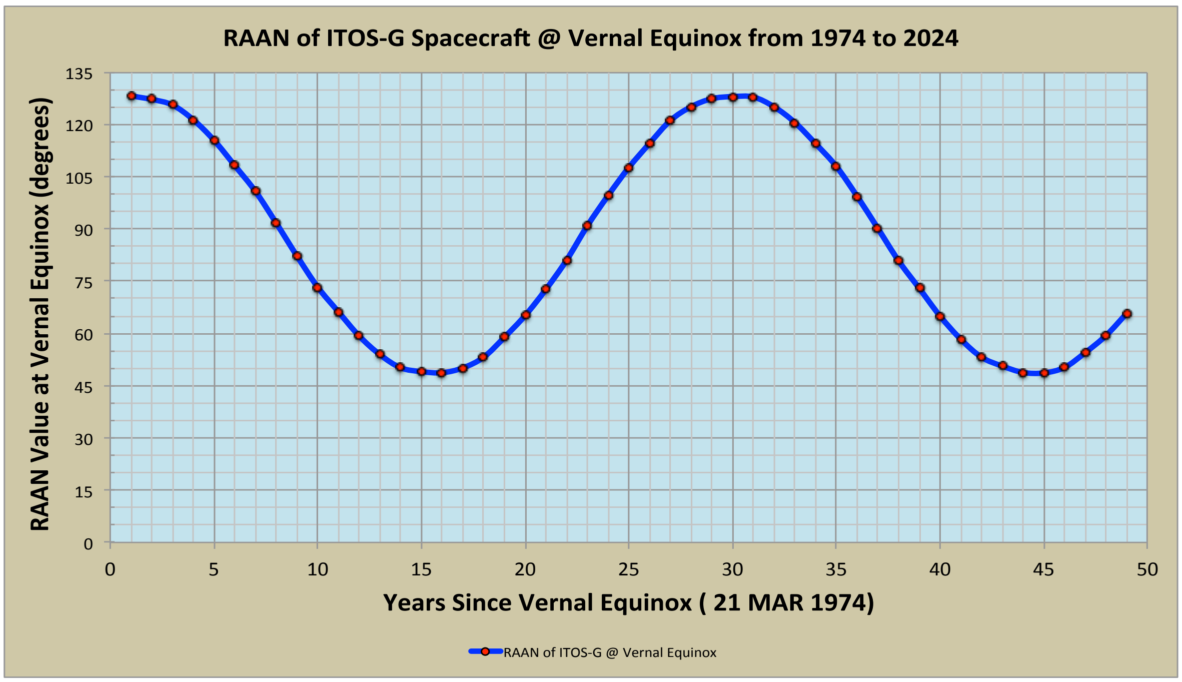

Figure AP1-8: RAAN Variations in ITOS-G Orbit at Vernal Equinox; 1974-2023

We decided to repeat the RAAN and inclination exercise for a 50-year period for the Object 7529, ITOS-G/NOAA-4. Using the same method we plotted the RAAN and inclination on Julian Day 80.0000 for the years 1974-2023. We present here, these long-term orbit parameters for that object, AO-7's co-passenger ITOS-G as Figure 8 and Figure 9.

Figures AP1-8 and AP1-9 should look familiar now. We see virtually no difference between these two figures and Figures AP1-4 and AP1-5. There is no difference in orbital performance here, based on variations in spacecraft mass or surface area - none at all. The derivative of the plot in Figure AP1-8 is a constant multiplied by the plot in Figure AP1-9.

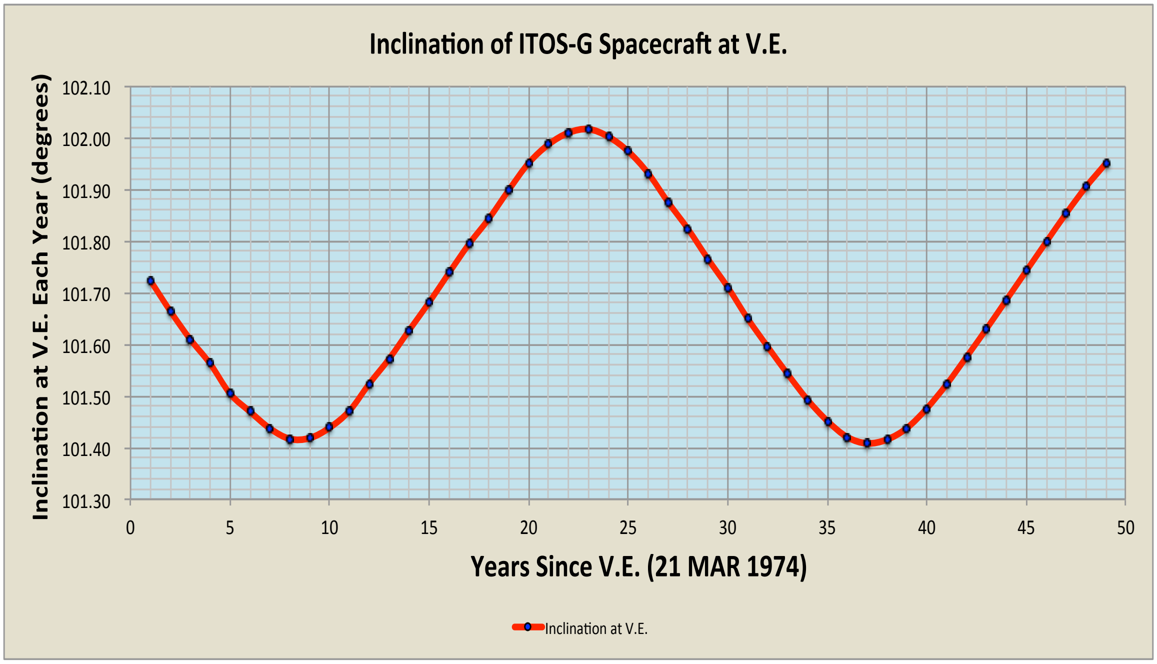

Figure AP1-9: Inclination Variations in ITOS-G Orbit at Vernal Equinox; 1974-2023

We therefore conclude that the forces acting on both bodies are the same force and we conclude that these variations can only come from an external force; not from within the Spacecraft/Earth orbital system. We leave it as a truly fun exercise for the readers of this paper to demonstrate that the orbits for INTASAT (Object 7531) and the Delta-104's 2nd Stage (Object 7532) have indeed been subjected to the same forces and have experienced the same outcomes. We next proceed with our analysis of the root cause for this rather long period orbit perturbation. See Appendix 2.

7.0 <u>*References:*</u>

1. Boain, Ronald J., *A-B-Cs of Sun-Synchronous Orbit Mission Design*, Doc. Id 20210001902, Pasadena, CA, Jet Propulsion Laboratory, National Aeronautics & Space Administration, Feb 1, 2004.

2. Wertz, James R., *Space Mission, Engineering: The New SMAD*, Space Technology Library, Microcosm Press, pp. 221-223 (2011).

3. Op. Cit., 1., Slide 14

<u>APPENDIX 2:</u> Physical Explanation of the Observed Oscillation

1.0 <u>Introduction:</u>

To understand the physical cause of the observed oscillation, it is helpful to note that the effect occurs in a system rotating once per year. This rules out most of the effects defined in inertial space or a system rotating with the earth – these effects would average out. Only the sun is essentially stationary with respect to the orbital plane of the sun-synchronous orbit. This makes it most likely that the observed effect is directly caused by the sun.

In principal there are two candidate processes, which could perturb the orbital plane. Because the orbit may be partially in shadow, asymmetrical light pressure could produce a torque on the plane.
Also the gradient of the solar gravity field could produce a torque on the orbital plane if the satellite at times is periodically closer or further away from the sun than the earth-center.

To decide which of the two effects is more likely to cause our observed oscillation, a rough analysis was done to compare the forces produced by light pressure and the forces by the solar gravity gradient. It turned out that the light-pressure forces were about two orders of magnitude smaller than the gravity gradient forces. Thus only the gravity gradient (GG) was investigated in our study.

If this problem is to be treated precisely, some rather complex spherical geometry equations need to be numerically integrated. This will lead to precise results, but will not necessary provide insight into the physical causes of the effect. Since we were mostly interested in understanding the cause of the observed 29-year period, we instead used some drastic simplifications to allow a simple physical analysis.

2.0 <u>The simplified approach:</u>

To make the geometry simple we assume for the analysis of the forces that the orbit is exactly polar, that the orbit is exactly circular, that the sun is always in the equatorial plane and moves with constant speed during the year. Furthermore, that we are only interested in the GG-induced torque-component in the polar direction; only this component changes the inclination (i) in the way required to explain the observed effect. See Fig. AP2-1 to visualize this geometry.

At first we compute the torque produced by the GG on the angular momentum of the orbit. (In the following formulas all angles are in radians and time is in seconds.)

The GG force acting on the spacecraft, when it is located at the ascending node on the equator and counted positive in the direction away from the sun, is given by:

$$F_G(\Phi) = 3n_e^2\, m\, d = 3n_e^2 m\, r\, sin\Phi \tag{1}$$

n_e is the mean motion of the earth-orbit in radians/s ,
(for the earth-orbit around the sun $n_e = 1.991*10^{-7}$ rad/s)
d is the radial distance of the satellite from the nominal orbit radius around the sun.
It is counted positive in the direction away from the sun.
m is the mass of the spacecraft
r is the radius of the spacecraft orbit
Ω Right Ascension of the s/c ascending node, RAAN
Φ is the sun-synchronous orientation angle, defined as $\Phi = \Omega - \Psi_s - \pi/2$
($\Phi = 0$ for the twilight orbit with RAAN being over 18:00 hours local time.)
Ψ_s is the mean Right Ascension of the sun;

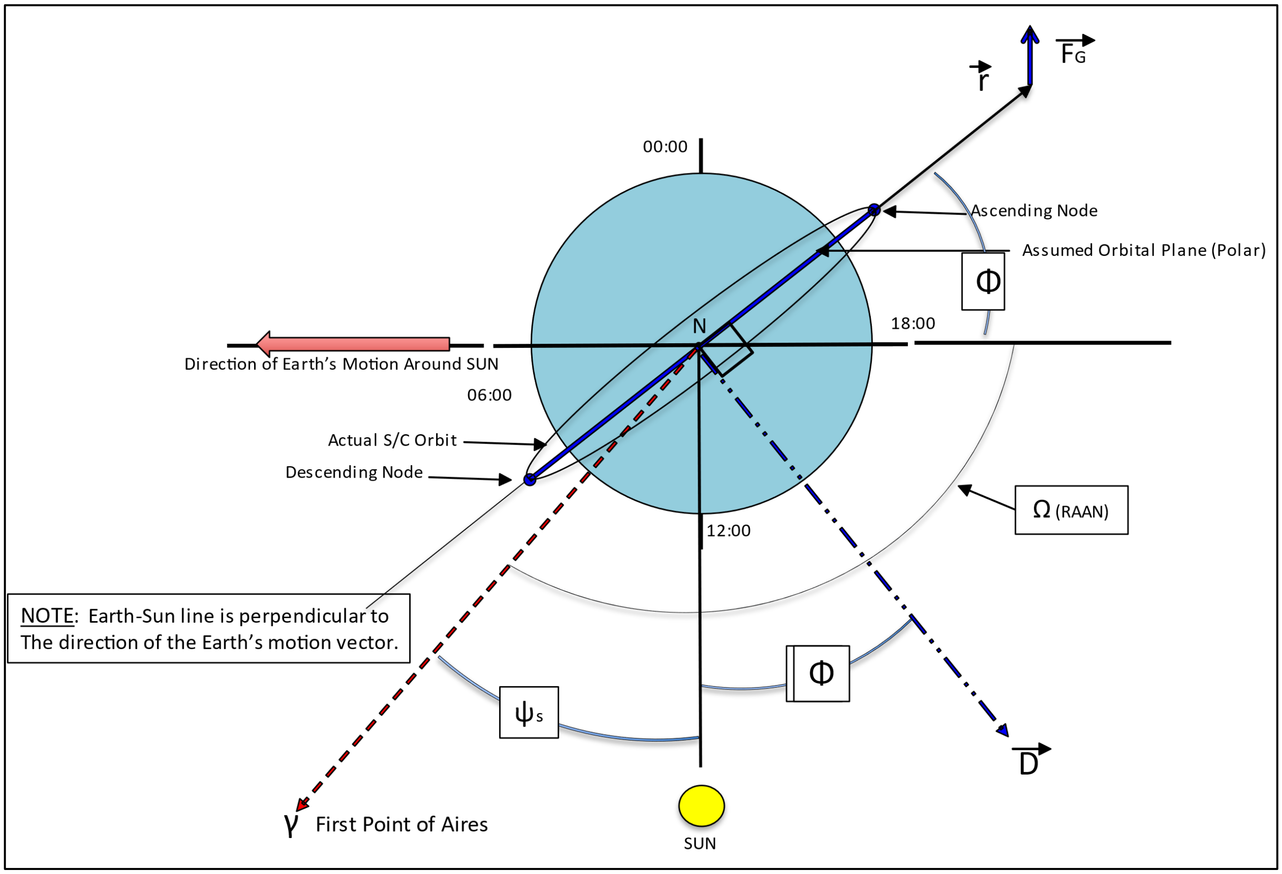

Figure AP2-1: Simplified Geometry as Seen from the North

(Sun-synchronous means that $d\Omega/dt$ is nominally equal to $d\Psi_{s/}/dt$ to keep Φ essentially constant.) The component of r in the equatorial plane and the force produced by the solar gravity gradient, also assumed to be in the equatorial plane, result in a north-pointing torque component T_p of the vector T.

$$\vec{T} = \vec{r} \times \vec{F_G} \qquad T_p = r \cdot \frac{F_G}{2} \cdot \sin(90 - \Phi) = r \cdot \frac{F_G}{2} \cdot \cos\Phi \tag{2}$$

The factor of ½ results from the fact that the spacecraft is not stationary at the equator, but moves around in the polar orbit, giving both the r-component in the equatorial plane and a factor of cos(τ) (τ = true anomaly) . And the average of $\cos^2(\tau)$ over one full orbit is ½.

With (1) and the angular momentum vector D of the orbit assumed to lie in the equatorial plane we obtain:

$$T_p = \frac{3}{2} \cdot n_e^2 \cdot m \cdot r^2 \cdot sin\Phi \cdot cos\Phi = \frac{d\vec{D}}{dt}, \frac{d\vec{D}}{dt}\ pointing\ north$$
$$\vec{D} = n_s \cdot I = n_s \cdot m \cdot r^2, Note: n_s = \omega(of satellite orbit) \tag{3}$$

As mentioned, we are interested only in the torque component pointing north, this torque component is changing the orbital angular momentum D such as to effect an inclination change. See Fig. AP2-2 for the geometry.

$$\frac{1}{\vec{D}} \cdot \frac{d\vec{D}}{dt} = \frac{3}{2} \cdot \frac{n_e^2}{n_s} \cdot cos\Phi \cdot sin\Phi = -\frac{di}{dt} \quad (4)$$

For our analysis we want this to be an essentially linear function of Φ, so we write

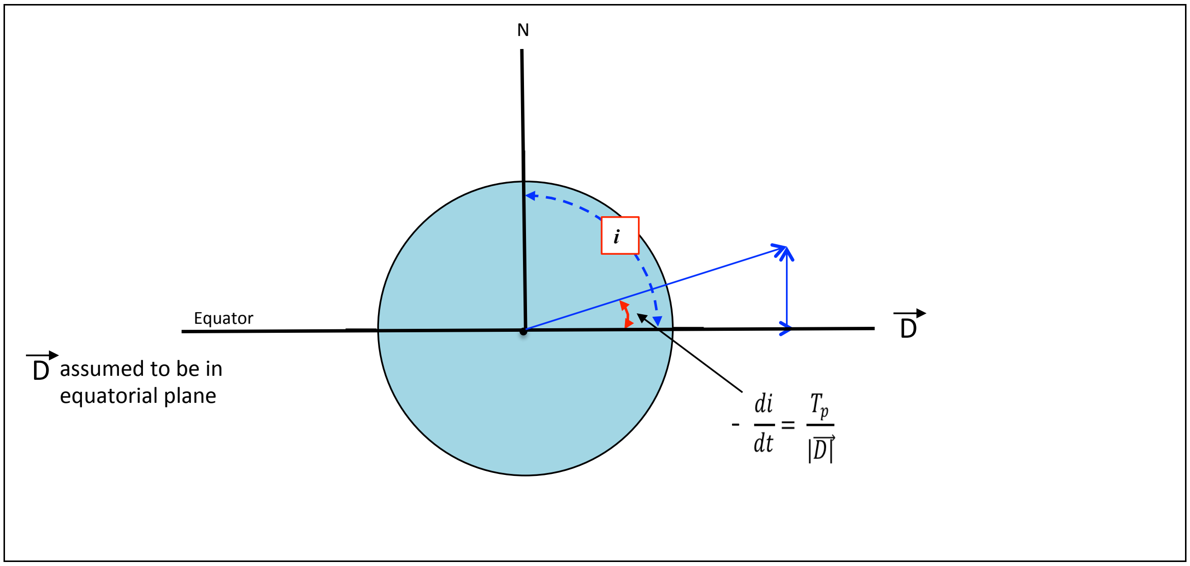

Figure AP2-2: The -di/dt Geometry
(Simplified looking in the equatorial plane from the descending node)

$$\frac{di}{dt} = -\frac{3}{2} \cdot \frac{{n^2}_e}{n_s} \cdot p(\Phi) \cdot \Phi = -H \cdot \Phi \; with \; p(\Phi) = \frac{sin(\Phi) \cdot cos(\Phi)}{\Phi} \quad (5)$$

The non-linearity of this equation is now contained in p(Φ) , which is close to 1 for small Φ. With Φ = 45°, -di/dt reaches a maximum and p(Φ) = 0.637 . With larger Φ both p(Φ) and -di/dt go down and for Φ = 90° they become zero. To get first order results we will take p(Φ) = 1; later on we will explore the impact of this step.

With the orbit of AO-7 we get for H = $6.5229 \cdot 10^{-11}$ s^{-1}

The orbit of AO-7 has been made sun-synchronous by adjusting the inclination of the orbit giving dΩ/dt a value to make it equal to the sun Right Ascension moving around the earth once per year. The well-known relation from perturbation theory gives dΩ/dt as:

$$\frac{d\Omega}{dt} = -\frac{3}{2} \cdot J_2 \cdot \sqrt{\frac{\mu}{R^3}} \cdot \frac{1}{\left(\frac{a}{R}\right)^{7/2} \cdot (1-e^2)^2} \cdot cos(i) \quad (6)$$

With an inclination larger than 90°, dΩ/dt becomes positive and thus can follow the daily increase of the sun-right ascension. We are now nearly finished; by taking the i-derivative of (6) we obtain the i-derivative of Φ in the vicinity of the nominal i producing the constant dΩ/dt for sun-synchronism:

$$\frac{d\cdot\left(\frac{d\Phi}{dt}\right)}{di} = \frac{3}{2}\cdot J_2\cdot\sqrt{\frac{\mu}{R^3}}\cdot\frac{1}{\left(\frac{a}{R}\right)^{7/2}\cdot(1-e^2)^2}\cdot sin(i) = M\cdot sin(i) \quad (7)$$

For the AO-7 orbit M = $9{,}8271\cdot 10^{-7}$ rad/s

3.0 The Harmonic Oscillator:

Multiplying (7) with (5) we obtain

$$\frac{di}{dt}\cdot\frac{d\cdot\left(\frac{d\Phi}{dt}\right)}{di} = \frac{d^2\cdot\Phi}{d^2t} = \ddot{\Phi} = -H\cdot M\cdot sin(i)\cdot\Phi \quad (8)$$

This is the classical differential equation of a harmonic oscillator with the solution:

$$\Phi = A\cdot cos(\varphi_0+\omega\cdot t) \quad \text{(a)}$$
$$\frac{d\Phi}{dt} = \dot{\Phi} = -\omega\cdot A\cdot sin(\varphi_0+\omega\cdot t) \quad \text{(b)}$$
$$\frac{d^2\Phi}{d^2t} = \ddot{\Phi} = -\omega^2\cdot A\cdot cos(\varphi_0+\omega\cdot t) \quad \text{(c)} \qquad (9)$$

(a) and (c) together give: $\ddot{\Phi} = -\omega^2\cdot\Phi$ (d)

By comparing (8) with (9 d) we get: $\omega^2 = H\cdot M\cdot sin(i)$ or

$$\omega = \sqrt{H\cdot M\cdot sin(i)} \quad (10)$$

For AO-7 we obtain $\omega = 7.9227\cdot 10^{-9}$ rad/s, which corresponds to an oscillation period of

P_0 = 25.13 years.

This is slightly faster than the observed 29 years. The most dramatic simplification of our model was the linearization introduced by taking p(Φ) = 1 in (5). In fact our result can be improved by observing that the non-linearity of a mathematical pendulum is p(α) = sin(α)/α , and by noting that the non-linearity of (5) can be written as p(Φ)= sin(2Φ)/2Φ . This is the same non-linearity as with the pendulum, only with the angle Φ required to be half as large as with a pendulum. Solving the pendulum equation with the non-linearity leads to an elliptical integral. The solution results in an increase of the period by a factor f; it is given using the, Arithmetic Geometric Mean' as

$$f = \frac{1}{AGM[1, cos(\varphi_{max})]}$$

With the observed Φmax = 40° of AO-7, we compute f = 1.1375, resulting in a corrected period of

P_1 = 28.58 years.

Despite our crude modeling this result is within a few percent of the observed period. This result gives us confidence that the observed 29-year oscillation is indeed caused by a solar gravity gradient orbit perturbation.

The variation of i

The analysis so far implies that the 29-year periodicity of Φ is primarily caused by the variation of inclination due to the solar GG. So there will be a fitting variation of i.

From (6) with M from (7) we get

$$\frac{d\Omega}{dt} = -M \cdot cos(i) \tag{11}$$

dΩ/dt consists of two parts, the variation of dΦ/dt and the constant $(d\Omega/dt)_{ss}$ to keep the orbit sun-synchronous. The dΦ/dt part as a function of time is caused by Δi(t):

$$\frac{d\Phi}{dt}(t) + \left(\frac{d\Omega}{dt}\right)_{ss} = -M \cdot cos\big(i_{ss} + \Delta i(t)\big) \tag{12}$$

Since Δi(t) is small, the cosine can be expanded as

$$\frac{d\Phi}{dt}(t) + \left(\frac{d\Omega}{dt}\right)_{ss} = -M \cdot cos(i_{ss}) + \Delta i(t) \cdot M \cdot sin(i_{ss}) \tag{13}$$

The condition for sun-synchronicity

$$\left(\frac{d\Omega}{dt}\right)_{ss} = -M \cdot cos(i_{ss}) \tag{14}$$

can be subtracted from (13) giving

$$\frac{d\Phi}{dt}(t) = \Delta i(t) \cdot M \cdot sin(i_{ss}) \tag{15}$$

Taking the first time derivative of our solution (9 b)

$$\frac{d\Phi}{dt}(t) = \dot{\Phi} = -\omega \cdot A \cdot sin(\omega t) \tag{16}$$

And combining (16) and (15) gives

$$-\omega \cdot A \cdot sin(\omega t) = \Delta i(t) \cdot M \cdot sin(i_{ss}) \quad (17$$

Solving for Δi(t)

$$\Delta i(t) = -\frac{\omega}{M \cdot sin(i_{ss})} \cdot A \cdot sin(\omega t) = -K \cdot A \cdot sin(\omega t) \quad (18)$$

With AO-7 we have K = $7.2\cdot10^{-3}$, $\omega = 6.9\cdot10^{-9}$ rad(s) and A = Φ_{max} = 40 deg, giving

$$\Delta i(t) = 0.288 \cdot sin(\omega t) \text{ in degrees} \quad (19)$$

This value is slightly smaller than the observed values of 0.3 deg · sin(ωt), which is caused by the non-linearity in our differential equation.

We now have the complete solutions for Φ(t) and Δi(t)

$$\begin{aligned} \Phi(t) &= A \cdot cos(\omega t + \varphi_0) \\ \Delta i(t) &= -A \cdot K \cdot sin(\omega t + \varphi_0) \end{aligned} \quad (20$$

The φ_0 allows to accommodate an initial injection error Δi_0 from the correct required i_{ss}.

Δi_0 = -K·A·sin(φ_0)

Dividing Δi(t) by Φ(t) and at injection time we have with ωt = 0 and $\Phi = \Phi_0$:

$$\frac{\Delta i_0}{\Phi_0} = -\frac{K \cdot sin(\varphi_0)}{cos(\varphi_0)} = -K \cdot tan(\varphi_0) \quad (21)$$

Solving for φ_0

$$\varphi_0 = arctan\left(-\frac{\Delta i_0}{K \cdot \Phi_0}\right) \quad (22)$$

And finally with (9) and the result of (22) we get for A

$$A = \frac{\Phi_0}{cos(\varphi_0)} \quad (23)$$